Application of Multi-Objective Optimization techniques to Geotechnical Engineering problems.

Thesis submitted in partial fulfillment for the award of degree of

MASTER OF TECHNOLOGY (DUAL DEGREE)

IN

CIVIL ENGINEERING

by

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MAY 2015

DEPARTMENT OF CIVIL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

CERTIFICATE

This is to certify that the thesis entitled **"Application of multi-objective optimization techniques to Geotechnical Engineering problems"** submitted by **Ankit Anand** to National Institute of Technology Rourkela, India for the award of degree of **Master of Technology (Dual Degree)** in engineering, is a bonafide record of investigation carried out by him in the Department of Civil Engineering, under the guidance of **Dr. Sarat Kumar Das**. The report is up to the standard of fulfilment of M. Tech (Dual Degree) degree as prescribed by regulation of this institute.

Dr. Sarat Kumar Das

Date:

Dept. of Civil Engineering

ACKNOWLEDGEMENT

First and foremost I would like to extend my sense of deepest gratitude and thanks to my guide and supervisor, **Dr. Sarat Kumar Das.** He was the guiding light and a source of inspiration during this wonderful journey of my first research work. Without him, this work would never have seen the shape it took finally and I admire him for the way he kept me encouraged always.

I also take this opportunity to sincerely thank the faculties and staff of Civil Department without whose valuable input, this work was not possible. The learning which took place in the department since first year helped me a lot to take this project to its final stage.

A good research work is never possible without a healthy discussion among our peers and for this, I will ever be obliged to my branch-mates who were an integral part of my project.

At last, I would like to thank my parents and my sisters for having belief and confidence in my abilities and thus enabling me to take up any challenge I faced during my work.

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ABSTRACT

This research work has its motivation in the ever-increasing use of computational methods in the areas of Civil Engineering. Parameter estimation has assumed a critical importance in predicting failure curves more accurately. Error-in-variables approach gives us a chance to predict simultaneously dependent and independent variables. A method like least square can take into account the error in only 'x' values and does not consider the error in values of 'y'. The vector of unknown parameters $(\sigma_c, \sigma_t, \emptyset)$ can also be estimated by the EIV approach along with the variable data points. The failure criterion used is the $MSDP_u$ rock failure criterion which deals with failure of low porosity rocks and represents a multi-axial surface in stress space.

The objective functions are modelled as a multi-objective optimization problem with the first function accounting for the error due to variables and the second function accounting for the error due to the model. Although, the optimization problem has increased dimension in case of EIV approach, it provides an efficient tool to predict the set of reconciled data and unknown parameters.

NSGA-II is an efficient MOEAs developed by Deb et al. (2001) for multi-objective optimization which follows the principle of a fast elitist non-dominated sorting procedure. The two error functions hence formulated by the EIV method is efficiently minimized by the evolutionary algorithm with a little bit of parametric tuning.

Estimating pile length for piles is quite difficult, and requires a good knowledge of the subsoil conditions. If the required conditions are formulated into objective functions along with constraint handling then optimized function of (d/L) against load bearing capacity can be found out by NSGA-II.

Keywords: Parameter estimation, rock-failure criteria, NSGA-II, optimization, EIV method, pile length.

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CHAPTER 1:
INTRODUCTION

1.1 Parameter Optimization of Rock Failure Criterion

The behavior of rocks plays an important role in engineering structures and various studies have been undertaken since the last few decades to analyze properties like stress-strain behavior, shear volume coupling, strength parameters of the rock sample and type of loading conditions. Rock engineering is a major part of Geotechnical engineering as we often come across structures where deep excavations have to be made in rocks. The ultimate state of stress or strain which causes failure has been the main focus of researchers, because in any construction the major aim is to avoid the failure state. Failure criteria is nothing but a mathematical equation to define the failure surface or curve. Over the years, numerous rock failure criteria have been given and there may be as many as 20 rock failure criteria (Yu, 2002). But every rock criteria is specific to a particular kind of rock. Thus, there has been constant efforts to modify and develop different failure criterion to apply to other rocks (Li et al. 2003). The failure criteria is aptly represented in a 2-D or 3-D space of stress or strain, which are predicted more accurately by rock strength parameters like uniaxial compressive strength, tensile strength, angle of internal friction etc. as these data are measured experimentally, there will always be some associated error based upon the precision of measuring equipment, non-homogeneity and anisotropy. Hence, these parameters need to be estimated more precisely by optimization techniques in order to better express the mathematical formulation of stress space. In order to find out such parameters, optimization methods are best suited (Desai and Chen 2006).

An appropriate objective function is chosen for the optimization procedure and it represents the gap between measured values and estimation needs. A statistical method is then generally used to formulate the objective functions. There are numerous statistical approaches for the formulation of required objective functions. Like for example, Parameter estimation in rock failure criterion is widely done by the method of least squares (Shah and Hoek 1992; Li et al. 2000) and weighted LS (Desai 2001). The drawback in these methods is that they create a distinction in independent and dependent variables (Das and Basudhar 2010). These methods make the assumption of independent variable being free from measurement errors. But there may be some kind of error in all the measuring parameters as measurement error is caused by various factors such as equipment, random testing effects and procedural.

The tri-axial test data for rock samples consist of the stresses (σ_1, σ_2, σ_3) and all these stress may be associated with some error based upon the accuracy of the experimental setup. A statistical method named as error-in-variables (EIVs) method has been suggested for use in optimization (Bard 1974; and Esposito and Floudas 1998) and is being utilized in various fields of engineering. In statistics, EIV method is considered as a regression model that accounts for measurement errors in independent variables unlike standard regression models that assume independent variables have been measured exactly and accounts only for errors in the dependent variables. The measurement errors in both dependent and independent variables after being taken into account, the true reconciled values of the variables can be predicted simultaneously by EIV method. But this leads to a larger dimension of the optimization procedure with the total number of parameters to be evaluated being x + y×z, where x = vector of model parameters, y = number of data points and z is the number of variables in each data set.

In the estimation and optimization of rock parameters, EIV method is used for formulation of objective functions to estimate rock strength parameters for multi-axial rock failure criteria. A multi-objective EA namely NSGA-II has been used for the optimization procedure and the failure criteria used for the present study is Mises-Schleicher and Drucker-Pager unified ($MSDP_u$) (Aubertin et al. 1999).

1.1.1 Error-in-Variables Approach

The main advantage of using error-in-variables approach is that in this method, a distinction is not made between dependent and independent variables. Both the system variables of x and y are assumed to be inclusive of error and objective function is formulated based on that assumption. EIV approach can be represented as:

$$f(\tilde{x}, \tilde{y}, £) = 0$$

Where £ is the vector of unknown parameter and $(\tilde{x}, \tilde{y})$ are true values.

1.1.2 Mises-Schleicher and Drucker-Pager unified ($MSDP_u$) Failure Criterion:

The $MSDP_u$ failure criterion is suitable for failure of low porosity rocks and represents a multi-axial surface in stress space (Aubertin et al. 1999). The same group had earlier developed a three dimensional criterion known as MSDP but unified it later, so that one doesn't have to deal with a transition condition while shifting from the curved portions (referred to as the Mises-Schleicher portion) and the linear portion (referred to as the Drucker-Prager portion). A failure envelope assumes importance as it delimits the stable and unstable zone. This delimitation is the basis of many practical applications (Zambrano-Mendoza et. al. 2002). A rock sample can be subjected to various tests in order to find out their peak strength. Loading conditions vary from uniaxial tension, diametrical compression, and reduced triaxial extension. The associated peak from these loading conditions from the stress-strain curve can be represented by a point that can be plotted in a two-dimensional space with appropriate combinations of (σ_1, σ_2, σ_3) or of the corresponding stress invariants (I_1, J_2, θ). A mathematical criterion involving these parameters is then conceived that tries to follow the curve represented by joining the points obtained through the various tests. $MSDP_u$ failure criterion has the following characteristics:

i. Tensile strength (uniaxial)
ii. Strength in compression
iii. A continuous influence of the minor principal stress for the entire range of behavior
iv. Strength when subjected to high confining stress, and
v. Loading geometry represented in the pi-plane.

The criterion is defined by the following equation:

$$\sqrt{J_2} - b\left\{\frac{\alpha^2(I_1^2 - 2a_1 I_1) + a_2^2}{b^2 + (1-b^2)sin^2(45° - 1.5\theta)}\right\}^{\frac{1}{2}} = 0$$

Where

$$\alpha = \frac{2 \sin \emptyset}{\sqrt{3}\,(3 - \sin \emptyset)}$$

$$a_1 = \frac{\sigma_c - \sigma_t}{2} - \frac{\sigma_c^2 - \left(\frac{\sigma_t}{b}\right)^2}{6\alpha^2(\sigma_c + \sigma_t)}$$

$$a_2 = \left\{ \left[\frac{\sigma_c + \frac{\sigma_t}{b^2}}{3(\sigma_c + \sigma_t)} - \alpha^2 \right] \sigma_c \sigma_t \right\}^{1/2}$$

And Lode angle (θ)

$$\theta = \tan^{-1} \frac{\sigma_1 + \sigma_3 - 2\sigma_2}{(\sigma_1 - \sigma_3)\sqrt{3}}$$

The above mathematical formulation of $MSDP_u$ failure criterion has been well verified with experimental results and validated over period of time. It differs from the most commonly used rock-failure criterion that is Hoek-Brown expression but represents well the short-term failure of various isotropic rocks.

1.2 Application of NSGA-II for optimization

The non-dominated sorting genetic algorithm (NSGA-II) was proposed by Deb et. al. (2001). It is freely available at the online GA laboratory named as KANGAL. NSGA-II characterizes fast non-dominated sorting procedure, a parameter less niching operator and an elitist-preserving approach. Its implementation process can be best described by following steps:

i. Initialization of a set of solution
ii. Non-dominated arrangement of solution in different fronts
iii. Crowding distance
iv. GA parameters like crossover and mutation.

CHAPTER 2:

LITERATURE REVIEW

2.1 Multi-objective algorithms applied for optimization and other problems in Civil Engineering:

MOEAs have been applied for optimization problems in civil engineering since the last decade. In practical problems, we often encounter situations where we have to deal with conflicting objectives where an optimal solution requires to consider trade-offs between the conflicting objectives. After a vast literature review it was found that tremendous amount of utilization of these algorithms have been done in water resource engineering and the other sectors of civil engineering have made use of the algorithms much less comparatively. The following paragraphs throws light on the amount of work done in different branches of Civil Engineering as regard to multi-objective optimization algorithms.

Structural Engineering

Lounis and Cohn (1993) presented a realistic and effective method to the optimization of pre-stressed concrete structures in case of two or more objectives must be satisfied at the same time. They treated the problem by using the most important objective function as the primary criterion and other remaining objectives were handled as constraints. Lagrangian algorithm was then used to solve the problem. Cheng and Li (1997) implemented a procedure incorporating a Pareto genetic algorithm (GA) and a fuzzy penalty function method to an example problem of multi-objective structural and control design of a truss which was a 72-bar space truss with two criteria, and a four-bar truss with three criteria. The results demonstrated that the proposed method is highly efficient and robust. The life-cycle maintenance and planning of deteriorating bridges was formulated by Liu and Frangopol (2005) as a multi-objective optimization problem that formulated life-cycle maintenance, safety conditions and other costs as separate objectives. An optimized trade-off between the conflicting objectives was found out and a diverse set of solutions representing maintenance scenarios was identified by the multi-objective GA used as a search engine. Further, Neves et. al.(2006) tried to identify the best maintenance plan for a group of bridges over a specified time horizon. A bridge manager generally deals with conflicting objectives, and this need was addressed by the authors in identifying a full probabilistic multi-objective approach to bridge maintenance considering single maintenance type. Marsh and Frangopol (2007) investigated optimization of cost and gap between corrosion rate sensors placed in bridge decks. The rate of corrosion of reinforcing steel can be extremely variable within a structure due to changing concrete properties with time, environmental fluctuations and numerous other factors. Corrosion rate sensors can help us to monitor the deteriorating conditions, but due to economic and construction constraints only a specified number of sensors can be used. A minimization problem was then formulated in order to minimize the cost of sensor system installation. The Pareto optimal fronts were obtained by using a multi-objective optimization algorithm in addition with interpolation techniques and Bayesian updating.

Transportation Engineering

The use of MOEAs has also been significant in this field as the practical problems faced by a transportation engineer is often multi-faceted in nature. Wang and Wright (1994) worked on interactive design of service routes. By its very nature, Transportation networks are complex and often involves conflicting, multiple objectives. Spatial network data, multi-objective optimization techniques and a user-friendly graphical interface was modelled by the authors. A genetic algorithm based procedure for handling multi-objective pavement maintenance programming problems was developed by Fwa et al. (2000). For better pavement maintenance planning and programming, analysis involving multiple considerations is required. Concepts of Pareto optimal solution set and fitness assignment based on ranks were adopted. A good analysis based on multiple handling of solution set and robust search characteristics indicated that genetic algorithms are suitable for multi-objective problems. Fan and Machemehi (2006) used a genetic algorithm to thoroughly analyze the characteristics on which an optimal bus transit route network design problem (BTRNDP) with variable transit demand is based upon. The multi-objective non-linear mixed integer model was formulated for BTRNDP consisting of three major attributes, a starting candidate route set generation method, a network analysis method and a genetic algorithm method that combined the former two components. A C++ program was written for the proposed solution procedure for the BTRNDP with variable transit demand. Abbas and Sharma (2006) used a multi-objective non-dominated sorting genetic algorithm for selecting the best timing plans which can be stored and grouped traffic states into a specified number of clusters, simultaneously associating every cluster with one of the timing plans. Optimal co-ordination of traffic signals required proper activation of timing plans in order to match an ongoing traffic conditions. A trade-off evaluation for multi-objective optimization in transportation asset management was done by Bai et. al. (2011).

To achieve an optimization at overall system level, a decision making process in transport asset management is characterized by a wide diversity of asset types. The analysis of trade-offs can be highly beneficial for these multi-objective problems. The objectives represented by considering network level performance measures was used to formulate the multi-objective optimization underlying the analysis of trade-offs. Then the extreme point NSGA-II was used to produce the Pareto frontiers that illustrated the trade-offs. Fwa and Farhan (2012) further explored the area of highway asset management to search for an optimal multi-asset maintenance budget allocation. In the allocation of maintenance budget, following key points are expected: (1) the preservation needs of all assets are effectively addressed, (2) the goals of distinct asset systems are optimally fulfilled in an impartial manner, and (3) the goals of the inclusive highway asset system are attained optimally. The outcomes were achieved by proposing a two-stage approach in which stage 1 consisted of generating a family of Pareto solutions and stage 2 adopted an optimal algorithm to distribute budget to specific assets by executing a cross-asset trade-off to attain optimal budget solution. Caetano and Teixeira (2013) proposed a multi-objective method for developing railway ballast, sleeper renewal operations and rail. Two objectives were considered for minimization: (1) railway track inaccessibility caused by railway track upkeep and renewal processes and (2) railway track components life cycle costs. A real case study was then taken up to study the numerical application of the model.

Construction Engineering and management

It involves designing, planning, construction and management of the whole process during the construction of an infrastructure. It usually involves a lot of conflicting objectives and as such MOEAs has been used considerably in this field too. One of the greatest challenges in this field is time-cost optimization (TCO), since optimization of one of the attributes endangers the other attribute. Most of the earlier studies focused on minimizing the total cost for an early completion of the project as found by Zheng et al. (2005). They proposed an integration of adaptive weight to represent the priority of fitness vector based on previous population set for their genetic algorithm. Similarly, safety and cost are another conflicting objectives whose trade-off investigation is of importance in construction engineering. An expanded site layout planning model was developed by Ei-Rayes and Khalafallah (2005) which was capable of increasing construction safety and at the same time decreasing the transportation cost of resources on location. A multi-objective genetic algorithm was then modelled taking into consideration all the decision variables as well as practical constraints into the site layout planning problem. In order to validate its capabilities in optimizing construction sites, a real world problem was then analyzed using the developed model. Kandil and proper and efficient utilization of resources is of critical importance in large-scale construction projects and for this purpose Ei-Rayes (2006) proposed a parallel GA framework. A multi-objective optimization module, a coarse-grained parallel GA module and a performance evaluation module was integrated to develop the parallel framework. The performance of the framework was evaluated using 183 experiments simultaneously implementing it on a cluster of 50 parallel processors. Khalafallah and Ei-Rayes (2008) also developed a model to minimize security risks during the construction of an airport expansion project. A multi-objective GA was then used to incorporate performance metrics and other criteria to investigate and increase the construction related security levels during an airport expansion. Orabi et. al.(2009) investigated the utilization of limited resources for reconstruction of damaged transportation networks. A heuristic model to optimize allocation of resources as well as simultaneously analyzing and evaluating the overall functional loss was developed to effectively plan reconstruction efforts. The model was developed through user symmetry algorithm and a multi-objective GA enabling the generation of optimal trade-offs between the two reclamation scheduling objectives. Zahraie and Tavakolan (2009) incorporated concepts of time-cost trade off and resource allocation in a multi-objective optimization model minimizing the total development time, resource moments and cost. NSGA-II was implemented for the optimization problem. Jun and Ei-Rayes (2011) presented the development of a fresh multi-objective optimization model that was capable of evaluating and minimizing unwanted resource fluctuations in construction scheduling.

2.2 Application of NSGA-II in Civil Engineering

Since its conception NSGA-II has been used in some fields of civil engineering for optimization. A comprehensive study of its application shows that the major fields in which it has been applied is water distribution systems, urban planning, environment and water resources and a few applications in the construction too.

The design of water distribution system (WDS) is a difficult and complex problem which involves a number of contradictory objectives. Until the advent of multi-objective evolutionary algorithms, most of the work focused on a single objective. NSGA-II and other evolutionary multi-objective algorithms made possible a scenario where a Pareto-optimal front could be achieved indicating trade-offs between different objectives. Two of the major objectives while designing such systems are system cost and benefit. Raad et. al. (2008) applied an evolutionary algorithm known as Jumping-Gene to a multi-objective WDS design and validated it by trying it on a number of recognized WDS yardstick. The results were compared with that found from NSGA-II. Preis and Ostfeld (2008) worked on optimal sensor design and NSGA-II was utilized to find the trade-off between various contradictory objectives. In the same year Weickgenannt et. al. optimized two objectives i.e (i) minimizing the risk of pollution and (ii) minimizing the number of instruments used as sensors using NSGA-II and determined that only a minor portion of a network is required to be monitored to deliver a good amount of safety. "Anytown" water distribution network is a complex system with a number of network elements. Prasad and Tanyimboh (2008) demonstrated the efficacy of the proposed model involving "Anytown" using NSGA-II considering two objectives of minimization of network cost and maximization of flow entropy. Behzadian et. al. (2008) compared two approaches of sampling design for the purpose of calibrating water distribution system model namely, stochastic and deterministic using NSGA-II to identify the whole Pareto-optimal front of optimal solution. Kanta et al. (2009) evaluated the conflicting objectives during the times of a hazard such as urban fire in order to ensure a reasonable delivery of water during both normal and emergency circumstances. Their goals also included maintaining good water standards and finding an economical rehabilitation procedure.

Environmental impacts of a WDS associated with the manufacture of water network components and to the generation of electricity for pumping water need optimum attention. Herstein and Filion (2010) devised a method that combined economic life cycle maintenance analysis with NSGA-II to decrease capital cost, energy use, and ecological power objectives. Residual chlorine control is an important aspect of water quality optimal control. Gao et al. (2010) tried to formulate objective functions for minimization of residual chlorine and WDS operational cost. Fu and Kapelan (2010) investigated the usage of artificial neural network in conjunction with genetic algorithms (GA) to improve computational effectiveness in solving multi-objective WDS design problems. Sensor placement problems were still a subject of research in large networks and as such Pinzinger et. al.

(2011) applied three algorithms (namely, integer linear programming and the other two based on the Greedy paradigm) to real case networks and compared the results of these algorithms based on these algorithms with an algorithm based on NSGA-II. They found out that the applied algorithms were better than NSGA-II in every single case. NSGA-II has also been combined with other techniques to give more robust solution. Wang et. al. (2012) combined cuckoo search with NSGA-II to generate a multi-objective cuckoo search (MOCS) algorithm.

Wang et. al. (2004) applied various multi-objective evolutionary algorithm (MOEAs) to solve the optimal design problems of WDS. The main purpose of this research work was to set a point of reference as no consistent effort were previously made to examine and report the finest known approximation of the true Pareto front (PF) for a set of yardstick difficulties.12 design problems collected from literature were used and 5 MOEAs were applied without due consideration given to parameterization in order to set the reference points. After the evaluation was done using 5 MOEAs, and Pareto-optimal set found out for all the design problems, it was concluded that no method was superior to the other. Even then, NSGA-II appeared to be a good choice as it generally required minimum parameter tuning and showed the best achievement across all the problems. Mala-Jetmarova et. al.(2014) investigated the trade-offs between water quality and pumping costs objectives of regional multi-quality WDS. They formulated a minimization problem with the concerned objectives. It was established that water efficacies which operate regional multi-quality non-drinking WDS can gain assistance from the investigation of trade-offs between water quality and pumping costs for the goal of operational planning. The same authors in 2015 also explored the influence of water-quality standards in source reservoirs for obtaining an optimal performance of a reginal multi-quality WDS.

Ozcan-Deniz et al.(2011) investigated the time, cost and environmental effect on construction operation optimization using genetic algorithms. Due to rising environmental concerns, it was necessary to incorporate environmental criterion along with other construction variables to model an effective eco-friendly construction management project. Using NSGA-II in MATLAB, they were successful in designing a framework in which project cost, time and environmental impact was of primary importance. The research in this field has gained much importance as sustainable construction has become critical to the architecture-engineering or construction industry. Inyim et.

al. (2014) used NSGA-II to find diverse solutions conforming conflicting objectives when it comes to sustainable development.

A construction project involves a lot a variables that can change the cost and duration of activities such as weather, resource availability, etc. resource leveling and allocation strategies also influence total time and costs of projects. Zahraie and Tavakolan et. al.(2009) tried to embed two concepts of time-cost trade-off and resource levelling and allocation in a stochastic multi-objective optimization model which minimizes the total project time, cost, and resource moments. They proposed a time-cost-resource utilization optimization (TCRO) model in which time and cost variables were considered to be fuzzy to increase flexibility for decision makers when using the model outputs. NSGA-II was used for the optimization problem and it was seen that in the two case studies taken, the Pareto front solutions of the TCRO model cover more than 85% of the ranges of total time and costs of solutions of the bi-objective time-cost optimization (TCO) model.

NSGA-II has also been used for justifiable planning in transport as the performance of multi-mode transport systems need to be analyzed carefully.. Khoo and Ong et. al. (2014) realized this need for proper planning of exclusive bus lane as it decreases road volume for non-bus traffic and can aggravate traffic cramming in urban cities. They incorporated a bi-objective optimization model to evaluate a bus lane plan taking into deliberation bus and non-bus traffic criteria. NSGA-II was implemented to solve the optimization model with a microscopic traffic simulation model. Their analysis showed that the proposed model was capable of producing exclusive bus lane schedules to release traffic cramming and simultaneously being sensitive to important parameters such as population size, travel request level and least duration of application. Deb et al. (2012) used NSGA-II for analyzing the stability of geosynthetic-reinforced embankments resting on stone-column-improved ground.

CHAPTER 3:

METHODOLOGY

3.1 Working Principles of NSGA-II

Deb et. al. (2001) suggested NSGA-II which was an improved version of NSGA. NSGA was mainly criticized for its computational complexity of $O(MN^3)$, its non-elitist approach and for the need of requiring a sharing parameter. All of this was overcome in NSGA-II which used a fast elitist non-dominated sorting. Its elitist approach lets it preserve the best Pareto-optimal solutions and at the same time, the characteristics of crowding distance maintains diversity in the set of solutions. NSGA-II as an MOEA has a computational complexity of $O(MN^2)$ and is efficient in finding a diverse set of solutions which have great convergence near the true Pareto-optimal front in comparison to other elitist MOEAs.

NSGA-II works with both real and binary-coded variables and a range is defined for each of its variables. The initialization starts with a set of population on the basis of range and constraints. It can handle multiple objectives and the fitness value of each objective is then calculated. A non-dominated sorting takes place according to the fitness value assigned and the first set contains the solutions which are not dominated by any other solution in the complete set. In order to maintain a diverse population, crowding distance is calculated for each solution which measures a solution's proximity to its neighbors. Each objective function is then assigned a distance value. The boundary solutions are assigned a distance value of infinity while other solutions are assigned a distance value equal to the difference in the function values of adjacent solutions. This distance is also normalized. Parents of the next population are selected based on their ranks i.e a solution having a lesser rank gets selected. If two solution has the same rank then the one with greater crowding distance is selected. The flowchart representing the major steps in NSGA-II is shown in Fig.1 and the mechanism of sorting by NSGA-II and crowding distance sort is shown in Fig.2.

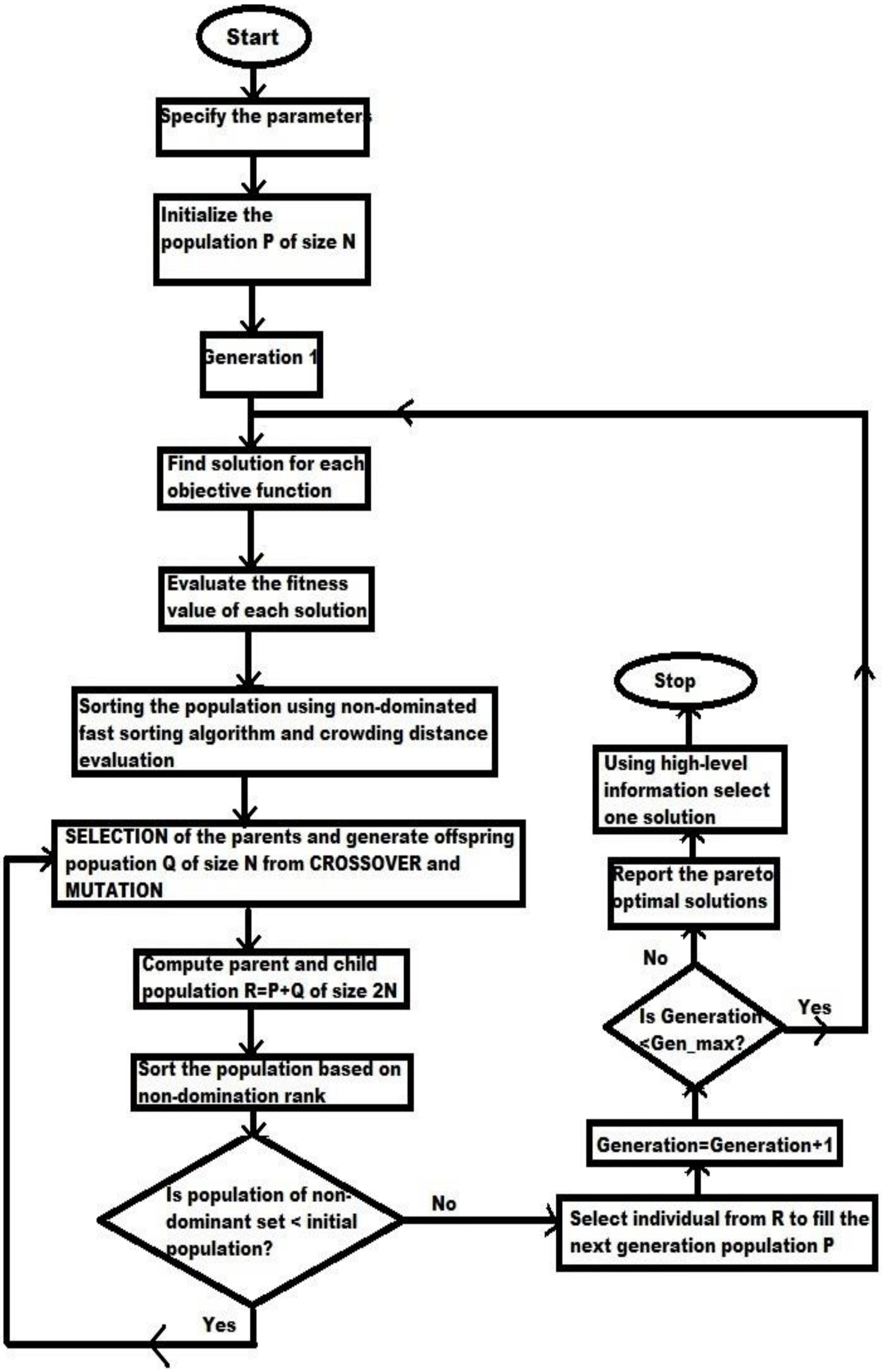

Figure 1: Flowchart showing the mechanism of NSGA-II

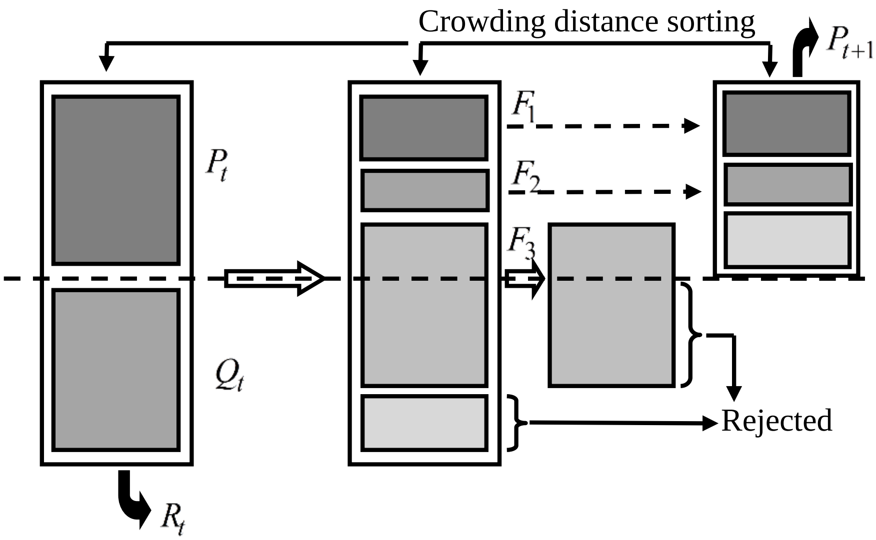

Figure 2:Non-dominated Sorting and crowding distance sorting

3.2 EXAMPLE ILLUSTRATING WORKING PRINCIPLES OF NSGA-II

A sample problem is taken to explain the working principle of NSGA-II. This is a two objectives optimization problem.

Min example:

$$\text{Minimize} \quad A_1(x) = y_1,$$

$$A_2(x) = \frac{1+y_2}{y_1}$$

Subject to $0.1 \leq y_1 \leq 1,\ 0 \leq y_2 \leq 5,$

The following GA parameters are used in the implementation of the algorithm:

Population Size ->6

Chromosome Length ->0

No. of generations ->50

No. of Functions ->2

No. of Constraints ->0

No. of real-coded variables ->2

Selection Strategy is Tournament Selection

Variable bounds are rigid

Crossover parameter in the SBX operator is 10.00

Cross-over Probability ->0.70

Mutation Probability for real-coded vectors -> 0.30

Random Seed ->0.65

The steps involved are elaborated in the following paragraphs. The different fronts are shown after 1st generation and finally the Pareto-optimal front after 50 generations is shown.

Table 1: Initial populations and off-springs with both the objective values

Parent population, P_t					Offspring population, Q_t				
Sol. No.	y_1	y_2	A_1	A_2	Sol. No.	y_1	y_2	A_1	A_2
1	0.2062	3.3873	0.2062	21.2744	a	0.4758	0.7893	0.4758	3.7610
2	0.4543	3.4728	0.4543	9.8463	b	0.8199	0.7185	0.8199	2.0959
3	0.4758	0.7893	0.4758	3.7610	c	0.9984	0.9752	0.9984	1.9784
4	0.8199	0.7185	0.8199	2.0959	d	0.2062	1.2502	0.2062	10.9114
5	0.5457	1.7956	0.5457	5.1233	e	0.4794	0.7154	0.4794	3.5783
6	0.9984	0.9752	0.9984	1.9784	f	0.4291	1.6874	0.4291	6.2623

Step1: $R_t = P_t \cup Q_t$ ={1,2,3,4,5,6,a,b,c,d,e,f}

In this step the parent and offspring populations are combined and R_t is created. A non-dominated sorting is performed on R_t and different fronts are identified.

F_1= {3, 4, 6, d, e, f}

Note that a, b & c in offspring solutions are similar to 3, 4 and 6 in parent solutions respectively.

F_2= {1, 2, 5}

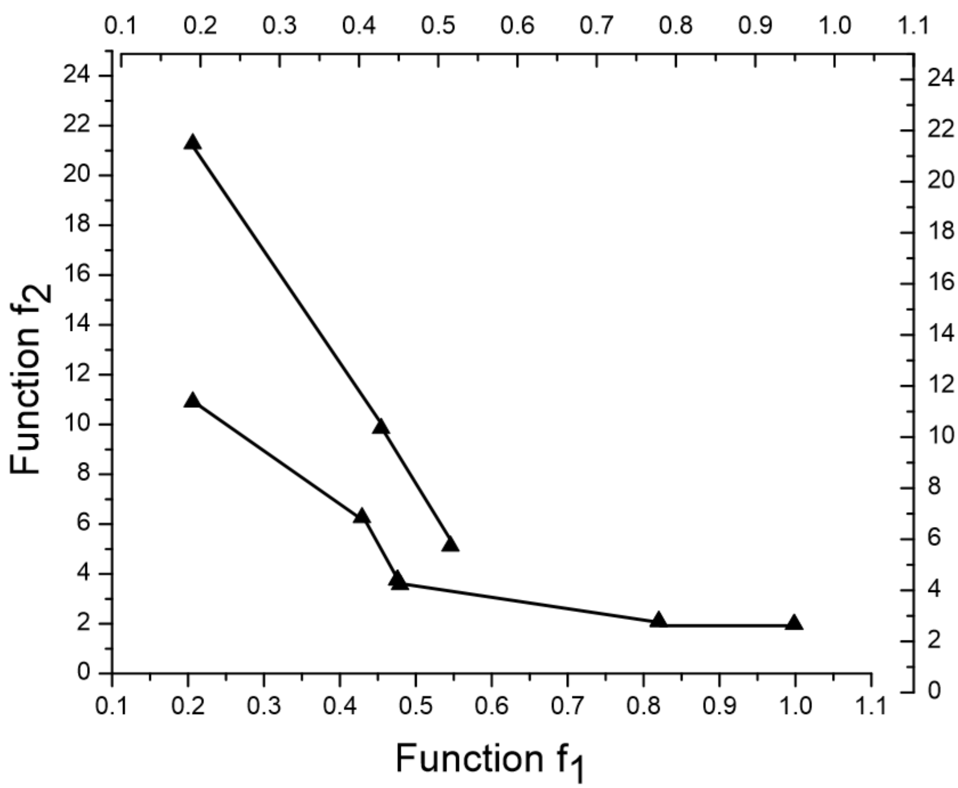

Figure 3: Pareto optimal set showing the two fronts of the combined population after 1st generation

Step2: Set P_{t+1}= {} and set counter i=1

Until $|P_{t+1}| + |F_i| < N$(=6, Population size), perform $P_{t+1} = P_{t+1} \cup F_i$ and i=i+1.

P_{t+1} = {3, 4, 6, d, e, f}

Here F_1 is sufficient for P_{t+1} as it has its members equal to population size i.e 6.

Step 3: From P_{t+1}, offspring population, Q_{t+1} is generated. It is done with the help of crowded tournament selection operator which compares two solutions and returns the winner of the tournament. It assumes that every solution I has two attributes:

i. A non-domination rank r_i in the population.
ii. A local crowding distance (d_i) in the population

A solution I wins the tournament if any of the following conditions are true:

i. If solution I has a better rank, that is, $r_i < r_j$.
ii. If they have the same rank but solution I has a better crowding distance than solution j, that is, $r_i = r_j$ and $d_i > d_j$.

Crowding Distance Assignment Procedure: Crowding –sort

Step C1: The number of solutions in F is called as L=│F│. For each I in the set, d_i is assigned as 0.

Step C2: For each objective function m=1, 2,…, M, the set is sorted in worse order of f_m.

Step C3: For m=1, 2,…, M, a large distance is allocated to the boundary solutions, or $d_{I_1^m} = d_{I_l^m} = \infty$, and for all other solutions j = 2 to (l-1), the crowding distance is assigned as

$$di_{j^m} = di_{j^m} + (f_m^{(I_j^{m+1})} - f_m^{(I_j^{m-1})}) / (f_m^{max} - f_m^{min})$$

By the above algorithm the crowding distance is calculated for each solution of P_{t+1}.

Table 2: The fitness assignment procedure under NSGA-II

Front1

Solution	x_1	x_2	f_1	f_2	f_1(sorting)	f_2(sorting)	Distance
3	0.4758	0.7893	0.4758	3.7610	third	fourth	0.1014
4	0.8199	0.7185	0.8199	2.0959	fifth	second	0.6038
6	0.9984	0.9752	0.9984	1.9784	sixth	first	∞
D	0.2062	1.2502	0.2062	10.9114	first	sixth	∞
E	0.4794	0.7154	0.4794	3.5783	fourth	third	0.4105
F	0.4291	1.6874	0.4291	6.2623	second	fifth	0.4207

For the given example the steps are implemented as follows:

Step C1: L=6 and $d_3 = d_4 = d_6 = d_d = d_e = d_f = 0$ is set. From the variable range we get,

$f_1^{max} = 1$, $f_1^{min} = 0.1$, $f_2^{max} = 60$ and $f_2^{min} = 1$.

Step C2: For the first and second objective function the sorting of the solutions is shown in Table A2 and is as follows:

1st objective function: {d, f, 3, e, 4, 6}

2nd objective function: {6, 4, e, 3, f, d}

Step C3: The solution 6 and d are boundary solutions for f_1. So we set $d_6 = d_d = \infty$. For other solutions, the calculations are done as follows:

$$d_3 = 0 + \frac{f_1(e) - f_1(f)}{f_1 \max - f_1 min} = 0 + \frac{0.4794 - 0.4291}{1 - 0.1} = 0.0559$$

$$d_4 = 0 + \frac{f_1(6) - f_1(e)}{f_1 \max - f_1 min} = 0 + \frac{0.9984 - 0.4794}{1 - 0.1} = 0.5767$$

$$d_e = 0 + \frac{f_1(4) - f_1(3)}{f_1 \max - f_1 min} = 0 + \frac{0.8199-0.4758}{1-0.1} = 0.3823$$

$$d_f = 0 + \frac{f_1(3) - f_1(d)}{f_1 \max - f_1 min} = 0 + \frac{0.4758-0.2062}{1-0.1} = 0.2995$$

The above distances are updated in the second objective. Again, $d_6 = d_d = \infty$ as they are boundary conditions. The other distances are calculated as follows:

$$d_3 = d_3 + \frac{f_2(f) - f_2(e)}{f_2 \max - f_2 min} = 0.0558 + \frac{6.2623-3.5783}{60-1} = 0.1014$$

$$d_4 = d_4 + \frac{f_2(e) - f_2(6)}{f_2 \max - f_2 min} = 0.5767 + \frac{3.5783-1.9784}{60-1} = 0.6038$$

$$d_e = d_e + \frac{f_2(3) - f_2(4)}{f_2 \max - f_2 min} = 0.3823 + \frac{3.7610-2.0959}{60-1} = 0.4105$$

$$d_f = d_f + \frac{f_2(d) - f_2(3)}{f_2 \max - f_2 min} = 0.2996 + \frac{10.9114-3.7610}{60-1} = 0.4207$$

Step 4: A sorting according to the descending order of the crowding distances obtained above yields the sorted set {6, d, 4, f, e, 3}. The offspring population is generated from this parent population. The exact offspring population will depend on the chosen pair of solutions participating in a tournament and the chosen crossover and mutation operators. For example if we pair solutions (6, 4), (d, f), (3,e), (d, 3), (4, e) and (f, 6), so that each solution participates in exactly two tournaments. In the first tournament, we observe that solutions belong to the same front. But $d_6 > d_4$, and hence solution 6 is a winner.

In the next tournament solution d wins. Performing other tournaments the mating pool is obtained which is {6, 6, d, d, e, 4}. Now these solutions can be mated pair-wise and mutated to create Q_{t+1}. This completes one generation of the NSGA-II.

Step 5: The parent population P_{t+1} {6, d, 4, f, e, 3} with its offspring population is shown below for 2nd generation.

Table 3: Parent populations and off-springs with both the objective values for 2nd generation

Parent population, P_{t+1}					Offspring population, Q_{t+1}				
Sol. No.	x_1	x_2	f_1	f_2	Sol. No.	x_1	x_2	f_1	f_2
1	0.4758	0.7893	0.4758	3.761	A	0.4758	0.7893	0.4758	3.7610
2	0.8199	0.7185	0.8199	2.095	B	0.8199	0.7185	0.8199	2.0959
3	0.9984	0.9752	0.9984	1.978	C	0.4291	1.6874	0.4291	6.2623
4	0.2062	1.2502	0.2062	10.91	D	0.2062	1.1937	0.2062	10.637
5	0.4794	0.7154	0.4794	3.578	E	0.4794	0.7154	0.4794	3.5783
6	0.4291	1.6874	0.4291	6.262	F	0.9984	0.9337	0.9984	1.9369

$F_1 = \{1, 2, 5, 6, d, f\}$

$F_2 = \{3, 4\}$

The process is repeated for P_{t+2} and Q_{t+2} as described for 2nd generation.

Step 6: The pareto-optimal front is found after 50 generations which is as tabulated below and the graph showing the front.

Table 4: Feasible and Non-dominated Objective Vector after 50 generations

S.No	Fitness Function, F_1	Fitness Function, F_2
1.	0.3611	2.7735
2.	0.1160	8.6249
3.	1.0000	1.0009
4.	0.1503	6.6543
5.	0.7697	1.3004
6.	0.1000	9.9981

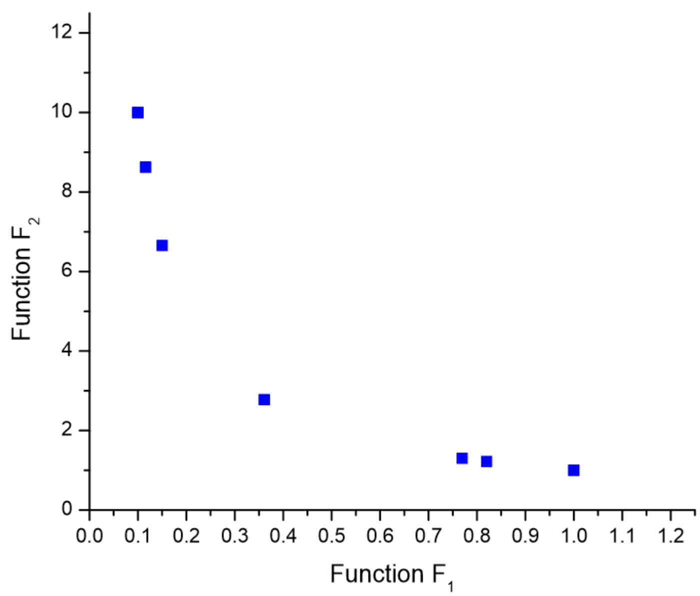

Figure 4: Pareto optimal front showing feasible and non-dominated objective vector after 50 generations.

CHAPTER 4:

PARAMETER OPTIMISATION OF ROCK FAILURE CRITERION

Parameter estimation has wide applications in various fields of civil engineering. Deb and Dhar (2011) developed a methodology for the identification of optimal design parameters for a system of beams resting on a stone column-improved soft soil.

4.1 EIV method

For the present study, variable points were taken from tri-axial tests of three rock samples namely, Trachyte, Dunham dolomite and dense marble. The goal of the EIV method is to minimize the error obtained between the observed and the reconciled values. The measured values can be formulated in terms of true values as below:

$$y = \tilde{y} + e_\mu \text{ , } \mu = 1, \ldots, n$$

where e_μ is the error in measured variables and $\tilde{y}$ is the true value.

The general equation for EIV approach is best represented by following equation:

In EIV, both the dependent and independent variables are estimated simultaneously and as such the dimension of optimisation problem increases. Total size of optimisation problem can be found out by following equation:

$$f(\tilde{x}, \tilde{y}, £) = 0$$

Total number of parameters = p + m × n

The $MSDP_u$ failure criterion involves three model parameters (£), namely $\emptyset$, σ_c and σ_t.

According to the above equation, the total number of parameters to be decided for each rock sample is as follows:

i. Trachyte with 31 data points: 3 + 31×3=96
ii. Dense Marble with 35 data points: 3 + 35×3=108
iii. Dunham Dolomite with 43 data points: 3 + 43×3=132

The concerned objective function is formulated as (Esposito and Floudas, 1998):

$$\text{Min} \sum_{\mu=1}^{n} \sum_{i=1}^{m} \left(\tilde{y}_{\mu,i} - y_{\mu,i}\right)^2 \qquad \text{.......(1)}$$

$$\text{Such that } f(\tilde{x}, \tilde{y}, £) = 0 \qquad \text{.......(2)}$$

4.2 EIV method as applied to $MSDP_u$ failure criterion

The rock failure criterion considered here was suggested by Aubertin et al. (1999) and stands for Mises-Schleicher and Drucker-Pager unified. It can be represented by the following equation:

$$\sqrt{J_2} - b\left\{\frac{\alpha^2(I_1^2 - 2a_1 I_1) + a_2^2}{b^2 + (1-b^2)sin^2(45° - 1.5\theta)}\right\}^{\frac{1}{2}} = 0$$

Where

$$\alpha = \frac{2\sin\emptyset}{\sqrt{3}\,(3 - \sin\emptyset)}$$

$$a_1 = \frac{\sigma_c - \sigma_t}{2} - \frac{\sigma_c^2 - \left(\frac{\sigma_t}{b}\right)^2}{6\alpha^2(\sigma_c + \sigma_t)}$$

$$a_2 = \left\{\left[\frac{\sigma_c + \frac{\sigma_t}{b^2}}{3(\sigma_c + \sigma_t)} - \alpha^2\right]\sigma_c\sigma_t\right\}^{1/2}$$

And Lode angle (θ)

$$\theta = \tan^{-1}\frac{\sigma_1 + \sigma_3 - 2\sigma_2}{(\sigma_1 - \sigma_3)\sqrt{3}}$$

It was necessary to have some constraints on the model parameters Das and Basudhar (2006) and Li et al. (2000) observed that following constraints holds good for the model parameters:

$$\sigma_c \geq 0,\ \sigma_t \geq 0, 0 \leq\ \emptyset \leq 60°, 5 \leq\ \frac{\sigma_c}{\sigma_t} \leq 50$$

4.3 OBJECTIVE FUNCTION

The MSDP$_u$ rock failure criterion is represented as follows in the EIV formulation:

$$\sum_{\mu=1}^{n} \left(J_{2\mu\ Experimental}^{1/2} - J_{2\mu\ Predicted}^{1/2}\right)^2 = 0$$

The above objective is satisfied must be satisfied by conditions represented in Eq. (1) and Eq. (2). A multi-objective error function is thus optimised using NSGA-II. The error functions are given as follows:

$$[\mathrm{ERR}(\mathrm{F}_1)] = \sum_{\mu=1}^{n} \sum_{i=1}^{m} \left(\check{y}_{\mu,i} - y_{\mu,i}\right)^2$$

$$[\mathrm{ERR}(\mathrm{F}_2)] = \sum_{\mu=1}^{n} \left(J_{2\mu\ Experimental}^{1/2} - J_{2\mu\ Predicted}^{1/2}\right)^2$$

The above mentioned error functions are minimised simultaneously for minimisation with the help of NSGA-II. The constraints on the model parameters are:

$$\sigma_c \geq 0,\ \sigma_t \geq 0, 0 \leq\ \emptyset \leq 60°, 5 \leq\ \frac{\sigma_c}{\sigma_t} \leq 50$$

The results were obtained for various error bounds in order to account for different ranges of error in the data points namely: ± 5%, ± 10% and ± 15%.

4.4 RESULTS AND DISCUSSION

Optimisation was done using a multi-objective evolutionary algorithm called non-dominated sorting genetic algorithm, NSGA-II. It uses a fast elitist non-dominated sorting approach to minimise objective functions. It can also maximize functions by using a negation (-1). Initialization of population takes with a set of specified population size. The algorithm starts working with a set of solutions rather than a single solution thus, giving a set of optimal solutions with each generation. The operators-crossover and mutation ensures that a diversified yet solutions close to true optimal Pareto solutions are preserved in each generation and are passed on as parent population for the next iteration.

The upper and lower bounds for variables were changed according to different error bounds. NSGA-II generates random values for the range specified and finds a better non-dominated set of solution with the last generation. It is also capable of constraint handling and the specified range for unknown parameters ($\emptyset$, σ_c, σ_t) were obtained using the constraints specified earlier.

The results obtained were compared with that obtained by Das and Basudhar (2010) using Genetic algorithm (Goldberg, 1989) and another evolutionary algorithm PGSL(Raphael and Smith 2003). The combined value of the two error functions obtained by NSGA-II is given in table 4.1 for Trachyte, Dunham dolomite and dense marble. It is seen that with increasing error bounds, the MOEA is able to better optimise the error functions with the least value for ± 15%.

Table 5: Optimal value of objective functions using NSGA-II for different rock samples

Rock Strength Data	Optimization methods	Optimum value of the objective function		
		Error bounds		
		± 5%	± 10%	±15%
Trachyte	GA	1,419.91	1,347.13	1,345.3
	PGSL	1,420.60	1,344.13	1343.43
	NSGA-II	1,431.32	1,387.27	1,353.06
Dense Marble	GA	504.32	447.35	420.18
	PGSL	503.13	447.29	419.44
	NSGA-II	505.60	456.71	432.32
Dunham Dolomite	GA	771.15	739.64	738.12
	PGSL	770.83	738.28	736.73
	NSGA-II	771.95	735.19	732.72

The results obtained by different EAs were comparable, with NSGA-II giving better results in case of Dunham Dolomite. Trachyte showed the highest value of error functions followed by Dunham Dolomite and dense marble. In case of GA and PGSL, only a slight difference in optimized values were found to be in case of ±10% and ±15%. But in case of NSGA-II, the difference was noticeable for Trachyte and dense marble.

4.4.1 Results obtained for Rock sample Trachyte

The graph of Pareto-optimal sets between the two error function is shown in Figure 4.1. The graph is plotted for error bound of ± 15%. The error function of $\sum_{\mu=1}^{n}\left(J_{2\mu\,Experimental}^{1/2} - J_{2\mu\,Predicted}^{1/2}\right)^2$ takes very high values when the error function F_1 is almost negligible.

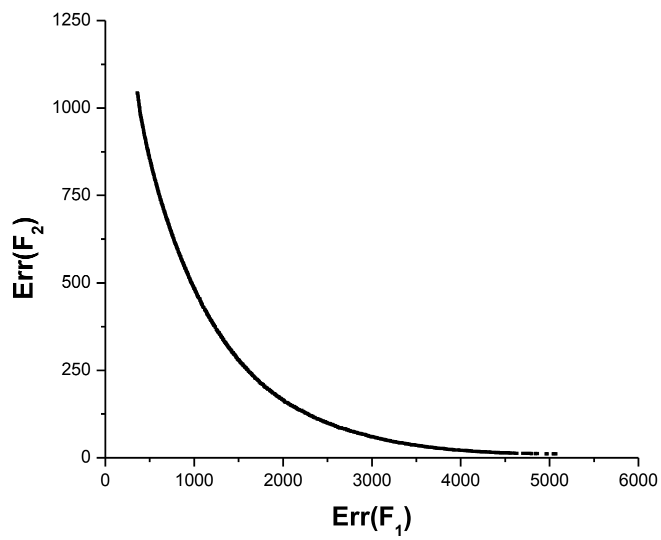

Figure 5: Variation of objective functions as given by NSGA-II for trachyte.

The vector of unknown parameters for different error bounds is given in table 4.2. The model parameter σ_c is found to vary in the range 123.47-139.43 in case of Trachyte, while the other two parameters of $\emptyset$ and σ_t were seen to vary in a range of 24.81°-25.49° and 2.49-13.99 respectively.

Table 6: Model parameters, error due to variables and error due to model for trachyte.

Parameters/errors	**Error Bounds**		
	± 5%	± 10%	±15%
Φ (deg)	25.49	25.36	24.81
$\boldsymbol{\sigma_c}$ **(MPa)**	123.47	123.50	139.43
$\boldsymbol{\sigma_t}$ **(Mpa)**	2.49	2.47	13.99
Errors due to variables	345.62	606.4109	535.25
Error due to model	1,085.70	780.8553	817.81
Total error	1,431.32	1387.266	1,353.06

The following pages contains the graphs of reconciled data and observed data in $J^{1/2} – I_1$ plane for different error bounds. It can be seen that $J^{1/2}$ reconciled values assumes higher values than the I_1 values.

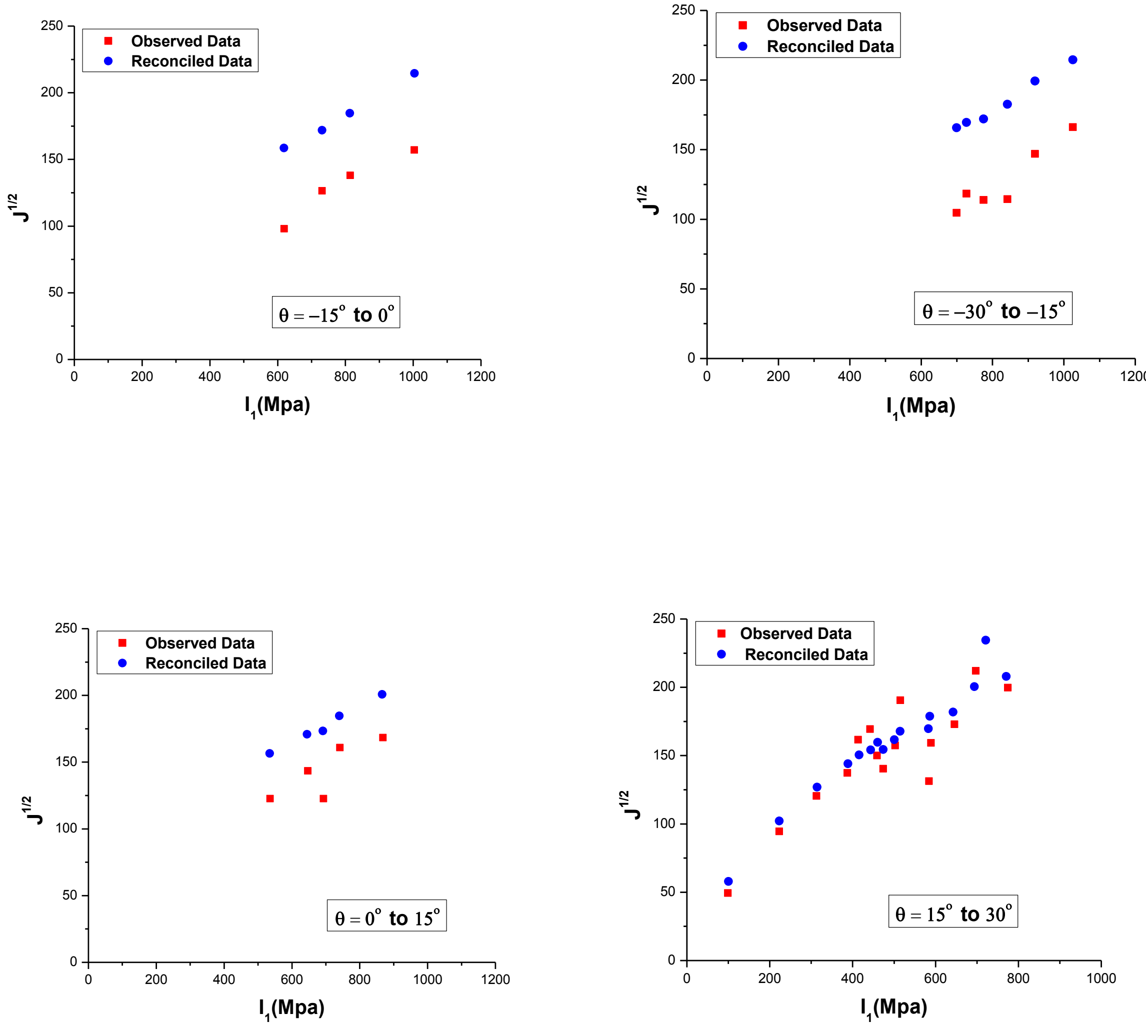

Figure 6: Observed and reconciled values using NSGA-II in $J_2^{1/2}$ - I_1 plane for ±5 % error bounds.

4.4.2 Results obtained for Rock sample Dense Marble

Figure 4.5 shows the variation of two error functions with the variation being same as that of Trachyte. Minimized values of the error functions are lower than that observed in case of Trachyte with the error function of $\sum_{\mu=1}^{n}\left(J_{2\mu\,Experimental}^{1/2} - J_{2\mu\,Predicted}^{1/2}\right)^2$ assuming highest value of 388.30 and the error function of $\sum_{\mu=1}^{n}\sum_{i=1}^{m}\left(\check{y}_{\mu,i} - y_{\mu,i}\right)^2$ having a highest value of 3585.98.

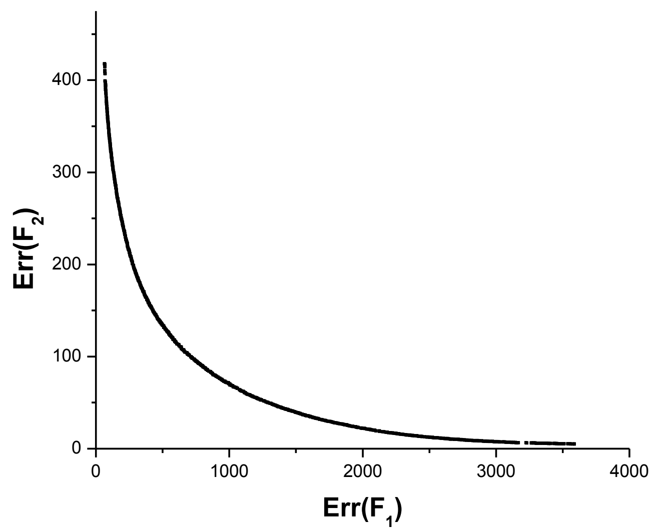

Figure 7: Variation of objective functions as given by NSGA-II for Dense Marble.

The vector of unknown parameters, and error functions due to variables and models for dense marble are given in Table 4.3. The model parameter σ_c is found to vary in the range 51.49-56.88 in-case of dense marble, while the other two parameters of ∅ and σ_t were seen to vary in a range of 38.21°-38.45° and 1.91-6.51 respectively.

Table 7: Model parameters, error due to variables and error due to model for Dense Marble.

Parameters/errors	Error Bounds		
	± 5%	± 10%	±15%
Φ (deg)	38.21	38.45	38.37
σ_c (MPa)	55.24	51.49	56.88
σ_t (MPa)	4.53	1.91	6.51
Errors due to variables	73.28	122.59	126.17
Error due to model	432.32	334.12	306.15
Total error	505.60	456.71	432.32

The graphs for reconciled data and observed data for various error bounds are given in the following pages. It is observed that the reconciled values and observed data in case of dense marble are in excellent agreement.

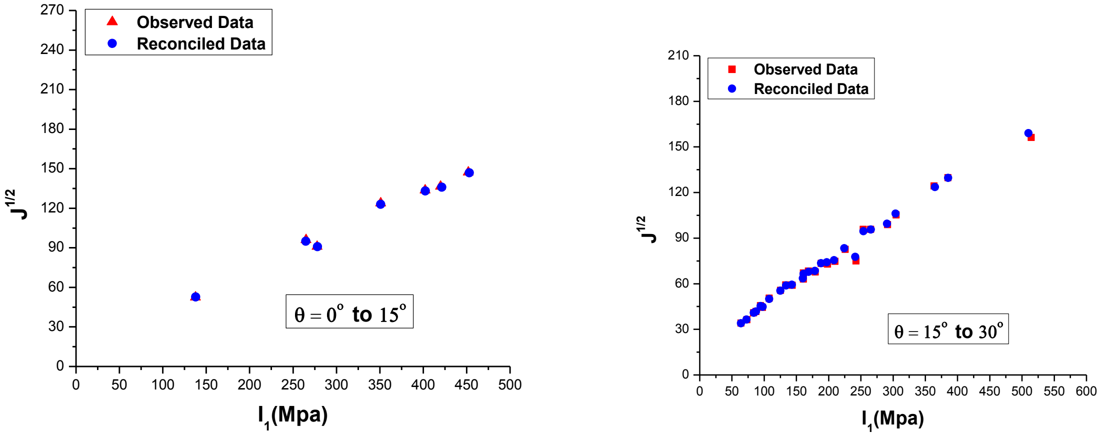

Figure 8: Observed and reconciled values using NSGA-II in $J_2^{1/2}$ - I_1 plane for ± 5 % error bounds.

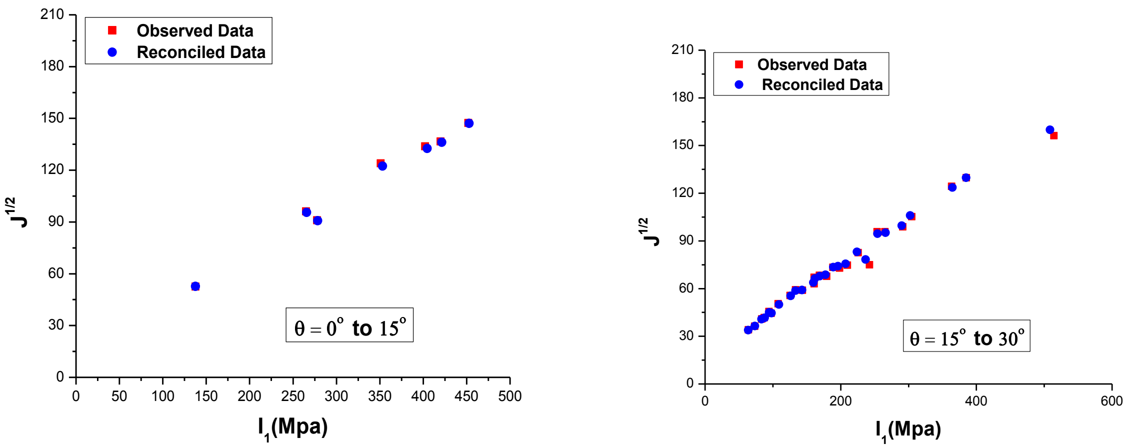

Figure 9: Observed and reconciled values using NSGA-II in $J_2^{1/2}$ - I_1 plane for ± 10 % error bounds.

4.4.3 Results obtained for Rock sample Dunham Dolomite

Figure 4.5 shows the variation of two error functions with the variation being same as that of other two rock samples. Minimized values of the error functions are lower than that observed in case of Trachyte with the error function of $\sum_{\mu=1}^{n}\left(J_{2\mu\,Experimental}^{1/2} - J_{2\mu\,Predicted}^{1/2}\right)^2$ assuming highest value of 505.72 and the error function of $\sum_{\mu=1}^{n}\sum_{i=1}^{m}\left(\check{y}_{\mu,i} - y_{\mu,i}\right)^2$ having a highest value of 4874.17.

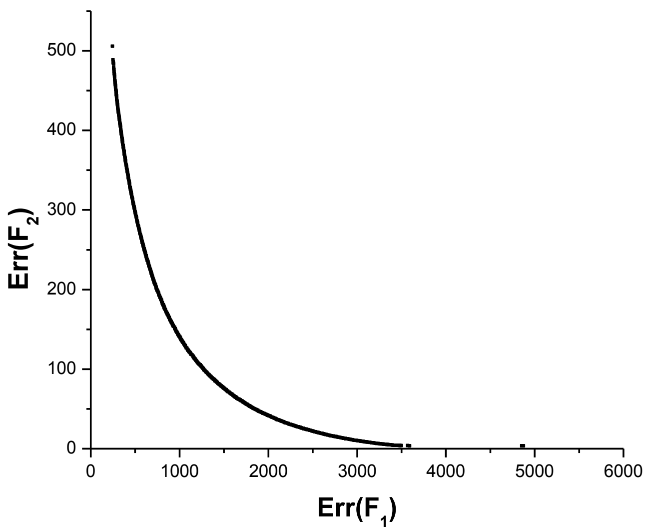

Figure 10: Variation of objective functions as given by NSGA-II for Dunham Dolomite.

The vector of unknown parameters, and error functions due to variables and models for dense marble are given in Table 4.4. The model parameter σ_c is found to vary in the range 313.47-317.72 in-case of Dunham dolomite, while the other two parameters of $\emptyset$ and σ_t were seen to vary in a range of 23.09°-23.51° and 10.28-12.20 respectively.

Table 8: Model parameters, error due to variables and error due to model for Dunham Dolomite.

Parameters/errors	**Error Bounds**		
	± 5%	± 10%	±15%
Φ (deg)	23.32	23.09	23.51
$\boldsymbol{\sigma_c}$ **(MPa)**	317.72	317.41	313.47
$\boldsymbol{\sigma_t}$ **(MPa)**	12.20	10.28	10.41
Errors due to variables	234.94	258.27	295.60
Error due to model	537.01	476.92	437.12
Total error	771.95	735.19	732.72

The following pages contains the graphs of reconciled data and observed data in $J^{1/2} – I_1$ plane for different error bounds. It can be seen that $J^{1/2}$ reconciled values assumes higher values than the I_1 values as in case of Trachyte.

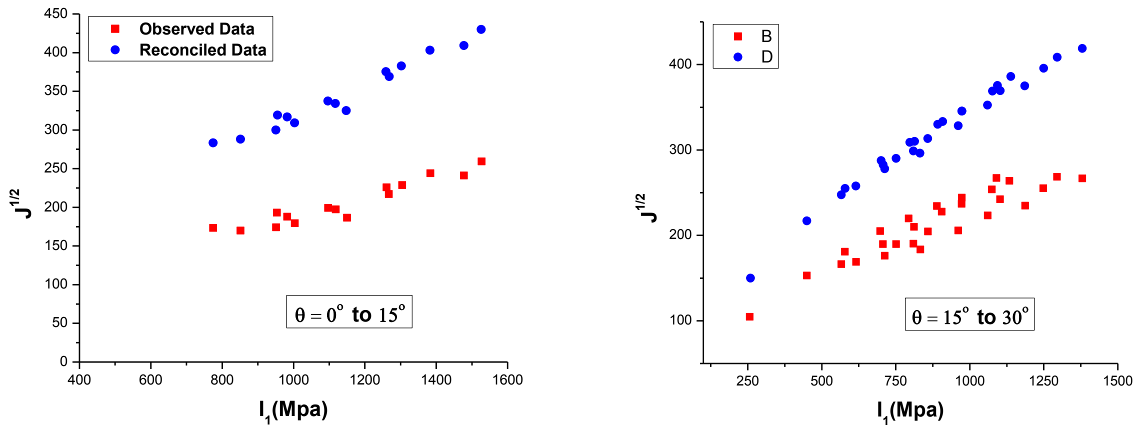

Figure 10: Observed and reconciled values using NSGA-II in $J_2^{1/2}$ - I_1 plane for ± 5 % error bounds.

The table in the following compares MSE values for the three rock samples using EIV approach for which optimization was done and the MSE values obtained by Das and Basudhar (2001).

Table 9: Comparison of MSE values for different error bounds.

Rock Sample	Optimization method	± 5%	± 10%	± 15%
Trachyte	NSGA-II	35.02	25.18	26.38
	PGSL	34.37	26.75	26.72
	LS		70.61	
Dense Marble	NSGA-II	12.35	9.55	8.75
	PGSL	12.23	9.56	8.22
	LS		20.09	
Dunham Dolomite	NSGA-II	12.48	9.69	10.16
	PGSL	13.54	11.59	11.55
	LS		26.36	

CHAPTER 5:

OPTIMISATION OF PILE FOUNDATION

5.1 Load Bearing capacity of a pile foundation in sand

Load bearing capacity of a pile foundation is derived from the frictional resistance(Q_s) provided along its length by the pile shaft and a pile point bearing capacity provided at the pile tip(Q_s). Numerous studies has been carried out in order to estimate the pile length and to predict the load bearing capacity accurately. Still, there is a lot of scope for investigation as the mechanisms involved are not yet fully understood and may never be. The analysis and design may thus be considered more of an art and it depends on the experience of the engineer, proper site investigation and good knowledge of the subsoil condition to successfully install a pile foundation. Piles are either installed in groups or individually depending upon the loading conditions of the superstructure. When installed in groups a pile cap is constructed to transfer the load of the superstructure to the pile foundation.

If there is a hard stratum, or there is a presence of bedrock the pile length can be estimated with good accuracy if a good record of subsoil conditions are present. In any case, the pile point resistance plays a good part in supporting the load.

Method for estimating Q_p (Meyerhof, 1976)

For piles in sand,

$$Q_p = A_p q_p = A_p q' N_q^*$$

There is a limiting value for Q_p as suggested by Meyerhof. The limiting point resistance, q_l is expressed by the following equation:

$$q_l = 0.5 P_o N_q^* tan\emptyset', \text{ and}$$

$$Q_p \leq A_p q_l$$

The value for N_q^* varies with soil friction angle and variation is given in Table 5.1 with different soil friction angle.

Table 10: Variation of N_q^* with soil friction angle (Meyerhof, 1976)

Soil friction angle, ∅ (deg)	N_q^*	**Soil friction angle, ∅ (deg)**	N_q^*
20	12.4	33	96.0
21	13.8	34	115.0
22	15.5	35	143.0
23	17.9	36	168.0
24	21.4	37	194.0
25	26.0	38	231.0
26	29.5	39	276.0
27	34.0	40	346.0
28	39.7	41	420.0
29	46.5	42	525.0
30	56.7	43	650.0
31	68.2	44	780.0
32	81.0	45	930.0

Frictional Resistance (Q_s) in sand

It is very difficult to estimate unit frictional resistance. It has been found that the unit frictional resistance increases up-to a certain depth and remains constant. This length is known as critical length.

Taking into account the above factor, the following relationship was developed:

For $z = 0$ to $z = L_{cr}$

$$f = K\sigma_o' tan\delta'$$

And for $z = L_{cr}$ to L

$$f = f_{z=L_{cr}}$$

Coyle and Castello (1981) proposed the following relation for Q_s

$$Q_s = K\overline{\sigma_o'}\tan(0.8\emptyset')\, pL$$

5.2 Application of NSGA-II for optimization of (d/L) ratio

In the present case, NSGA-II was applied for the objectives involved in order to get optimized length of a pile foundation. This is a simple implementation of a MOEA taking its advantage of obtaining diverse and non-dominated solutions to optimize geometrical dimension of a pile foundation. Although there are many criteria to decide the length of a pile foundation, but knowing the proper soil condition and other parameters, we can take advantage of evolutionary algorithms in order to get good estimation of design attributes like length.

Two objectives were formulated for the present case: the first one being of total load bearing capacity in sand, second of ratio of diameter of pile to its length (d/L). The first function is maximized as we need to have maximum load bearing capacity. The second objective of the ratio of design attributes of pile foundation is to be maximized too, as it will result in smaller lengths of pile foundation giving better load bearing capacity. In NSGA-II, we can maximize a function by multiplying it with (-1).

Formulation of objective functions:

i. Total Load bearing capacity: to be maximized

$Q_{total} = Q_p + Q_s$

where Q_p and Q_s are calculated according to the equations mentioned above.

ii. Ratio of diameter of pile foundation to its length (d/L): to be maximized

Subject to the constraint: L/d $\leq$ 20 and

$$Q_p \leq A_p q_l$$

5.3 Results and discussion

The diameter is varied from 600mm to 1000 mm with an interval of 100mm to see the variation of d/L against ultimate load bearing capacity. A population size of 100 is used with a total of 250 generations in NSGA-II.

The upper and lower bounds for length of the pile was taken to be from 4m to 40m.

The internal angle of friction for sand, $\emptyset$ is taken as 25°, dry unit weight is taken as 17kN/m^3 and for piles in sand, c’= 0.

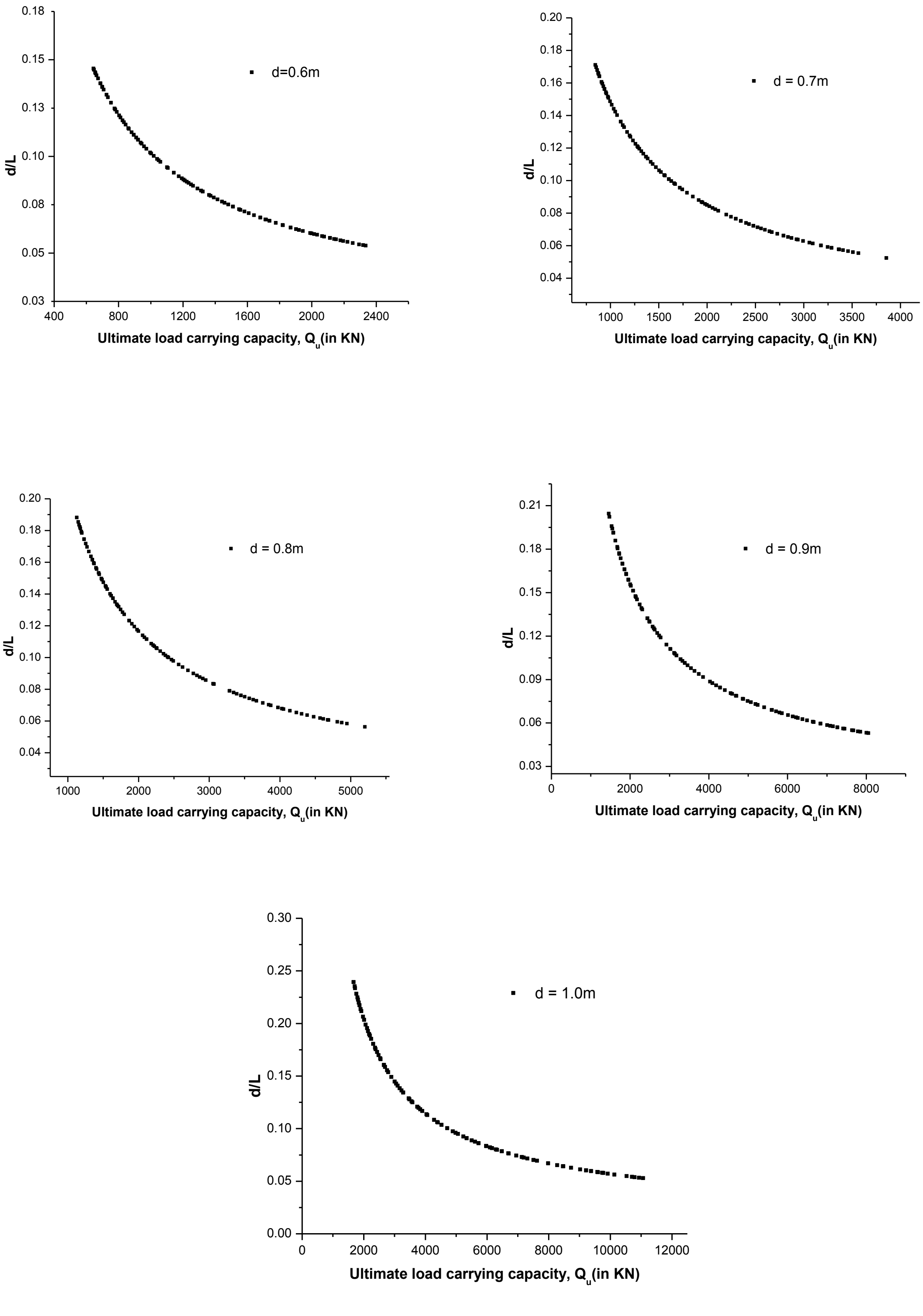

Figure 11: Variation of d/L versus Q_u for different values of d (friction angle=25°)

The above graphs show that with a constant diameter, if the length is increased the ultimate load bearing capacity will increase as suggested by decreased values of (d/L) ratio for higher Q_u. In Figure 17, the values for different diameter of pile foundation is compared. It is seen that the maximum diameter of 1.0m with minimum (d/L) ration can give us the maximum load bearing capacity.

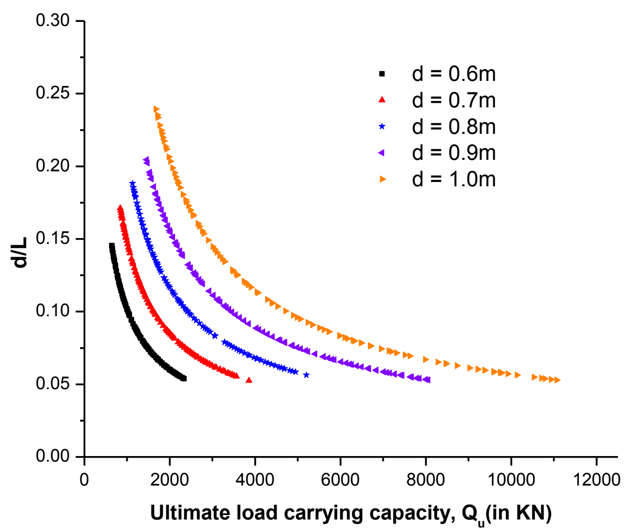

Figure 12: Comparison of d/L vs Q_u for various values of d (friction angle=25°).

The analysis of load bearing capacity with varying(d/L) ratio in earlier pages was done for an internal angle of friction, 25°. The same analysis was carried out with a friction angle of 30°. The results obtained are given in the following page.

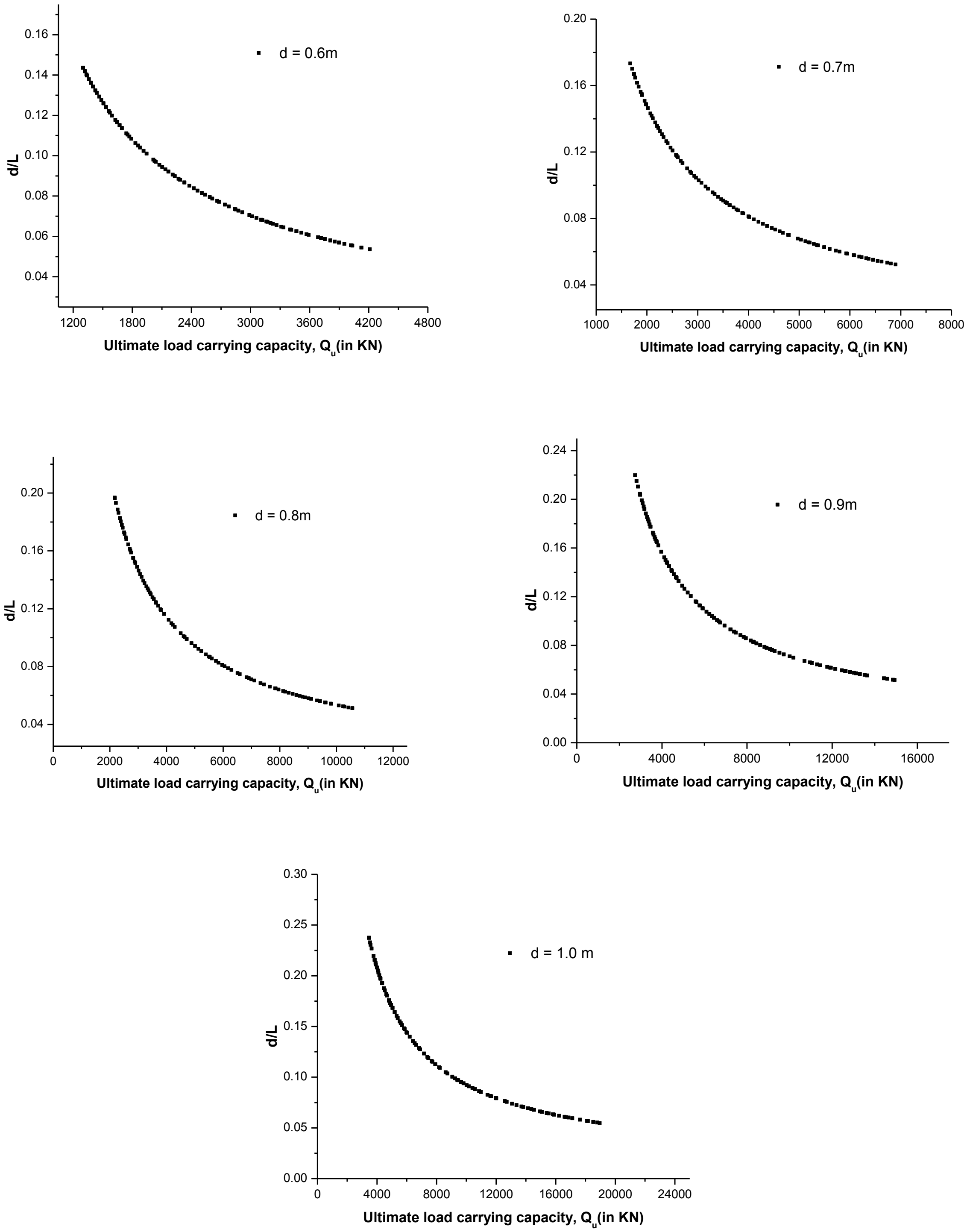

Figure 13: Variation of d/L versus Q_u for different values of d (friction angle=30°)

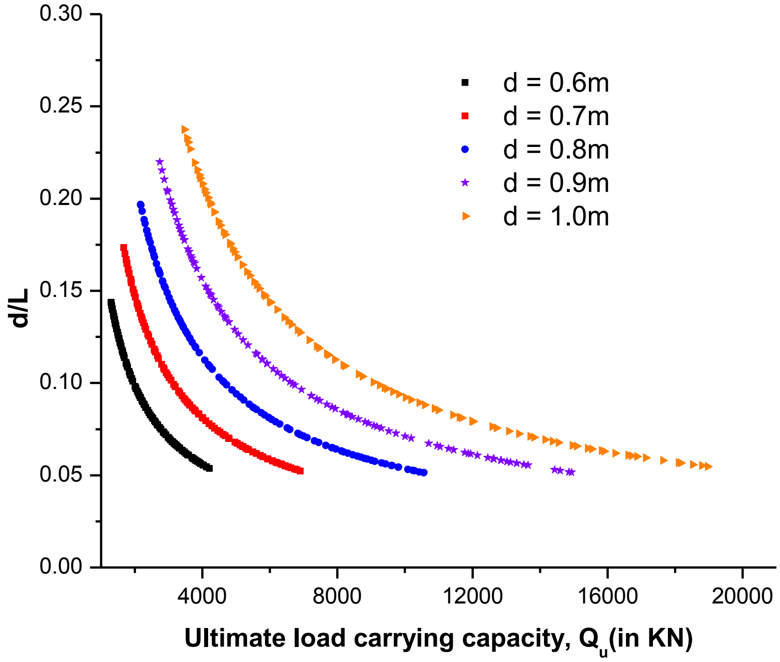

Figure 14: Comparison of d/L vs Q_u for various values of d (friction angle=30°).

The results for friction angle 25° and 30° were compared for the diameter of 1.0m in Figure 20. It was seen that an increase in friction angle results in a considerable increase in load bearing capacity of the pile foundation. Hence, with low (d/L) ratio and a higher friction angle the load bearing capacity can be maximized.

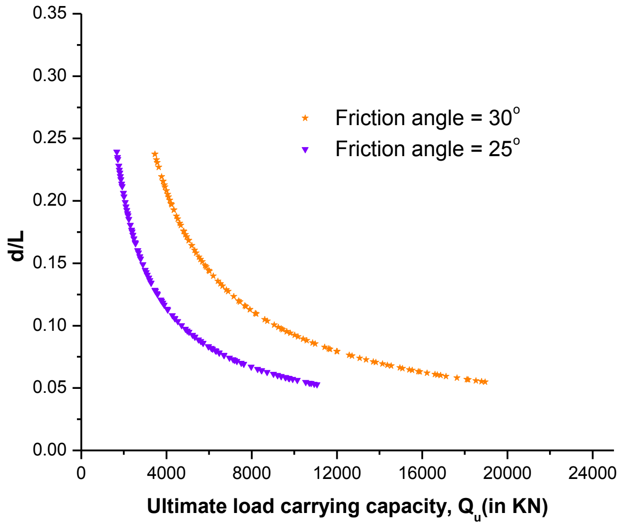

Figure 15: Comparison of (d/L) vs Q_u for different angle of internal friction.

CHAPTER 6:

CONCLUSION

6.1 Parameter optimization of rock failure criterion

The results obtained showed that error-in-variables approach is a good technique to implement for parameter optimization. Although the number of parameters increases in case of EIV approach, it gives a chance to reconcile both dependent and independent variables. The results were found to depend on the error bounds used (± 5%, ± 10%, ± 15%). The error function both for variables and for the model is seen to decrease with increasing error bounds. For example, in case of Dunham Dolomite, the combined optimized value for the error function was 771.95, 735.19 and 732.72 for ± 5%, ± 10%, ± 15% respectively.

From the table of comparison between MSE values for NSGA-II and PGSL, it is observed that the EIV method gives much improved results than the LS values. The MSE values for different rock samples and for different error bounds are in good agreement for NSGA-II and PGSL. In case of Dunham Dolomite, the MSE values given by NSGA-II are much better in all the cases of error bounds indicating that with good parameter input, it can outperform other MOEAs.

6.2 Optimization of Pile foundation

The graphs obtained by NSGA-II in this case shows that with increasing length with a fixed diameter of pile foundation provides good load bearing capacity. The best load-bearing capacity and length combinations are seen with lower (d/L) ratio. As the diameter is increased to a maximum of 1.0 m, we see that there is a considerable increase in load bearing capacity. The results for friction angle of 25° and 30° was also compared for a diameter of 1.0m. It is seen that a higher friction angle results in a considerable increase of maximum load bearing capacity at lower (d/L) ratios.

CHAPTER 7:

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